

University of Mumbai

Program: **Cyber Security**

Curriculum Scheme: Rev2019

Examination: SE Semester :III

Course Code: CSC302

Course Name: Discrete Structures and Graph Theory

Time: 2 hour 30 minutes

Max. Marks: 80

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks 02 marks each
1.	In how many ways can one go from $(-2; -2)$ to $(2; 2)$ without crossing $(0; 0)$, if in one step one can move either one co-ordinate horizontally or vertically ? (One only moves up and to the right and cannot come back.)
Option A:	70
Option B:	36
Option C:	48
Option D:	34
2.	An algebraic structure _____ is called a semigroup.
Option A:	$(S, *)$
Option B:	$(S, +, *)$
Option C:	$(S, +)$
Option D:	$(+, *)$
3.	Determine whether the following graphs are isomorphic. if yes select correct correspondence.
Option A:	no
Option B:	yes, $a \rightarrow a'$, $b \rightarrow b'$, $c \rightarrow c'$, $d \rightarrow d'$, $e \rightarrow e'$
Option C:	yes, $a \rightarrow a'$, $b \rightarrow c'$, $c \rightarrow e'$, $d \rightarrow b'$, $e \rightarrow d'$
Option D:	none of above
4.	Select correct definition for regular graph
Option A:	A simple graph of n vertices in which the degree of each vertex is $(n-1)$

Option B:	A graph in which every vertex has the same degree
Option C:	Drawing the diagram of a given graph in such a way that no two edges intersect
Option D:	none of above
5.	Check for additive group Z_4 and multiplicative group non zero Z_5
Option A:	is isomorphic
Option B:	is not isomorphic
Option C:	is homomorphic and Isomorphic
Option D:	none of above
6.	Let f and g be the function from the set of integers to itself, defined by $f(x) = 2x + 1$ and $g(x) = 3x + 4$. Then the composition of f and g is
Option A:	$6x + 9$
Option B:	$6x + 7$
Option C:	$6x + 6$
Option D:	$6x + 8$
7.	What equation would you form for “the product of three consecutive numbers is divisible by 6 ” to proof by Mathematical Induction.
Option A:	$n(2n)(3n)=6m$
Option B:	$n(n+1)(n+2)=6m$
Option C:	$n*m*p=6m$
Option D:	none of above
8.	For the statement “If I come early then I can get a car”. Select correct contrapositive statement.
Option A:	If I cannot get a car then I cannot come early
Option B:	If I cannot come early then I cannot get a car
Option C:	If I can get a car then I can come early
Option D:	None of above
9.	Let $A = \{1, 2, 3, 4\}$, and let $R = \{(1, 2), (2, 3), (3, 4), (2, 1)\}$. Find the transitive closure of R .
Option A:	$R = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,4)\}$
Option B:	$R = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,4), (4,3)\}$
Option C:	$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,4)\}$
Option D:	None
10.	Let the students who likes table tennis be 12, the ones who like lawn tennis 10, those who like only table tennis are 6, then number of students who likes only lawn tennis are, assuming there are total of 16 students.
Option A:	16
Option B:	10
Option C:	4
Option D:	8

Q2	Solve any Two Questions out of Three	10 marks each
A	Prove that the set $G = \{1, 2, 3, 4, 5, 6\}$ is a Finite Abelian group of order 6 with respect to multiplication modulo 7.	
B	<p>Let $A = \{a, b, c, d\}$ and x be a relation on A whose matrix is</p> $M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ <p>Prove that R is partial order. Draw Hasse diagram of R.</p>	
C	<p>Write Prim's Algorithm. Apply it to following graph.</p>	

Q3.	Solve any Four Questions out of Six	5 marks each
A	<p>Prove by Mathematical Induction method -</p> $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$	
B	Define group; monoid, semigroup:	
C	<p>Find out Hamiltonian path and Hamiltonian cycle.</p>	
D	$R = \{0, 2, 4, 6, 8\}$. Show that R is a commutative ring under addition and multiplication modulo 10. Verify whether it is field or not	
E	Let L be the bounded distributive lattice. Prove that if complement exist then it is unique	
F	If f is a homomorphism from a commutative semigroup $(S, *)$ onto a Semigroup $(T, *)$. Then prove that $(T, *)$ is also commutative.	

Q4.		
A	Solve any Two	5 marks each
i.	<p>Check whether relation is reflexive, irreflexive, symmetric, anti symmetric, transitive.</p> $R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 3), (3, 4), (4, 4)\}$ $R_2 = \{(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)\}$	

ii.	Define planar graph. What are the necessary and sufficient Conditions to exist Euler path, Euler circuit and Hamiltonian Circuit?
iii.	Let L be the bounded distributive lattice. Prove that if complement exist Then it is unique.
B	Solve any One 10 marks each
i.	Prove that if x is a rational number and y is an irrational number, then $x + y$ is an irrational number.
ii.	In any Ring $(R, +, \cdot)$ prove that i) The zero element z is unique. ii) The additive inverse of each ring element is unique.