

(Time: 3 hours )

Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks.

Q1 (a) Find Laplace transform of  $\frac{\cos\sqrt{t}}{\sqrt{t}}$  given that  $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-(1/4s)}$  [5]

(b) Calculate Spearman's rank correlation coefficient for the following data: [5]

X	32	55	49	60	43	37	43	49	10	20
Y	40	30	70	20	30	50	72	60	45	25

(c) Find inverse Laplace transform of  $\frac{2s-1}{s^2+8s+29}$  [5]

(d) If  $f(z) = qx^2y + 2x^2 + ry^3 - 2y^2 - i(px^3 - 4xy - 3xy^2)$  is analytic, find the values of p, q, and r [5]

Q2 (a) Find Laplace transform of  $e^{3t} f(t)$  where  $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$  [6]

(b) Two unbiased dice are thrown. If X represents sum of the numbers on the two dice. Write probability distribution of the random variable X and find mean, standard deviation, and  $P(|X-7| \geq 3)$  [6]

(c) Obtain Fourier series for  $f(x) = x \sin x$  in the interval  $0 \leq x \leq 2\pi$ . [8]

Q3 (a) Using Milne-Thompson's method construct an analytic function  $f(z) = u + iv$  in terms of z where  $u + v = e^x(\cos y + \sin y) + \frac{x-y}{x^2+y^2}$  [6]

(b) Using convolution theorem find the inverse Laplace transform of  $\frac{(s+3)^2}{(s^2+6s+5)^2}$  [6]

(c) Fit a parabola  $y = a + bx + cx^2$  to the following data and estimate y when  $x=10$  [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Q4 (a) Find Laplace transform of  $e^{-(1/2)t} t f(3t)$  if  $L\{f(t)\} = \frac{1}{s\sqrt{s+1}}$  [6]

- (b) Find half range sine series for  $f(x) = x - x^2$ ,  $0 < x < 1$ . [6]

Hence deduce that 
$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi^3}{32}$$

- (c) Given regression lines  $6y = 5x + 90$ ,  $15x = 8y + 130$ ,  $\sigma_x^2 = 16$ .  
Find i)  $\bar{x}$  and  $\bar{y}$ , ii)  $r$ , iii)  $\sigma_y^2$  and iv) angle between the regression lines [8]

- Q5 (a) Can the function  $u = r + \frac{a^2}{r} \cos\theta$  be considered as real or imaginary part of an analytic function? If yes, find the corresponding analytic function. [6]

- (b) An unbiased coin is tossed three times. If  $X$  denotes the absolute difference between the number of heads and the number of tails, find moment generating function of  $X$  and hence obtain the first moment about origin and the second moment about mean. [6]

- (c) Evaluate  $\int_0^\infty e^{-2t} \cos t \int_0^t u^2 \sinh u \cosh u \, du \, dt$  [8]

- Q6 (a) Find inverse Laplace transform of  $\frac{1}{(s-2)^4(s+3)}$  using method of partial fractions. [6]

- (b) If a continuous random variable  $X$  has the following probability density function [6]

$$f(x) = \begin{cases} k e^{-\frac{x}{4}}, & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases} \quad \text{find } k, \text{ mean and variance.}$$

- (c) Find half range cosine series for  $f(x) = x$ ,  $0 < x < 2$ . [8]

Hence deduce that i) 
$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \dots = \frac{\pi^4}{96}$$

ii) 
$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots = \frac{\pi^4}{90}$$

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(3 Hours)

Total Marks: 80

- N.B:** 1. Question No. 1 is compulsory  
 2. Attempt any 3 from remaining questions  
 3. Assume any suitable data if necessary and justify the assumptions

- Q.1 Attempt any **Four**. 20
- Give difference between random scan display and raster scan display.
  - Define Aliasing, Describe different antialiasing techniques.
  - Compare DDA and BRESENHAM line drawing algorithm.
  - Explain point clipping algorithm.
  - Give fractal dimension for KOCH curve.
- Q.2 a) Derive formula for mid-point circle algorithm. 10  
 b) Given a line AB where A(3,1) and B(0,0) calculate all the points of line AB using DDA algorithm. 10
- Q.3 a) With neat diagram explain Composite transformation. 10  
 b) Describe what is Homogeneous coordinates. 10
- Q.4 a) With neat diagram explain window to viewport coordinate transformation. 10  
 b) With neat diagram explain Sutherland Hodgman polygon clipping algorithm. 10
- Q.5 a) Define projection, with neat diagram describe planar geometric projection. 10  
 b) Describe properties of BEZIER curve. 10
- Q.6 a) Describe various principles of traditional animation. 10  
 b) Write short note on Depth buffer algorithm. 10

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(3 Hours)

Total Marks: 80

- N.B:** (1) Question No. 1 is compulsory.  
 (2) Attempt any three questions out of the remaining five questions.  
 (3) Figures to the right indicate full marks.  
 (4) Make suitable assumptions wherever necessary.

- Q.1 (a) Compare linear and non-linear data structures. [05]  
 (b) Explain the advantage of circular queue over linear queue. Write a function in C language to insert an element in circular queue. [05]  
 (c) Define binary search tree. Discuss the case of deletion of a node in binary search tree if node has both the children. [05]  
 (d) Write a C function to search a node in doubly linked-list. [05]

- Q.2 (a) Construct AVL tree for the following sequence: [10]  
 67,34,90,22,45,11,2,78,37,122  
 (b) Write algorithm for postfix evaluation. Demonstrate the same step by step for the expression:  $9\ 6\ 7\ * \ 2\ / \ -$  [10]

- Q.3 (a) Write a program to perform following operations on a circular linked list: [10]  
 i) insert a node from the end of the list, ii) delete first node,  
 iii) count the number of nodes with even values, iv) display the list.  
 (b) Write a C program to simulate linear queue as linked list. [10]

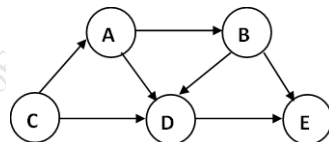
- Q.4 (a) Construct Huffman tree and find the Huffman codes for each symbol given below with frequency of occurrence: [10]

Symbol	p	g	e	r	i
Frequency	20	17	33	25	40

- (b) Explain the various ways to represent graph in the memory with example. [05]  
 (c) Construct binary search tree from given traversal sequences: [05]

In-order traversal	D	E	B	A	C	F	G	I	H	J
Pre-order traversal	F	E	D	C	B	A	G	H	I	J

- Q.5 (a) Apply linear probing to hash the following values in a hash table of size 11 and find the number of collisions: 67,44,90,12,83,52,23,87,79. [10]  
 (b) Define topological sorting. Perform topological sorting for the following graph: [10]



- Q.6 (a) Construct a B tree of order 3 by inserting the following given elements as: [10]  
 77,97,75,64,53,14,26,49,82,59.  
 Show the B tree at each step of insertion.  
 (b) Write a function in C for DFS traversal of graph. Explain DFS graph traversal with suitable example. [10]

- N.B. 1. Question No. 1 is compulsory  
 2. Attempt any **three** questions from remaining five questions  
 3. Assume suitable data if **necessary** and justify the assumptions  
 4. Figures to the **right** indicate full marks
- Q1 A Convert 05  
 i) 123 in to binary  
 ii)  $(AB9)_{16}$  in to Decimal  
 iii)  $(351)_8$  in to decimal  
 iv) 129 in to BCD  
 v) 64 in to gray code
- Q1 B Draw the single and double precision format for representing floating point number 05  
 using IEEE 754 standards and explain the various fields
- Q1 C Explain SR Flip Flop 05
- Q1 D Differentiate between Hardwired control unit and Micro programmed control unit 05
- Q2 A Draw the flow chart of Booths algorithm for signed multiplication and Perform 10  
 $5 \times 2$  using booths algorithm
- B Explain the different addressing modes. 10
- Q3 A For 132.65 obtain the IEEE 754 standards of Single precision and Double precision 10  
 format
- B Explain Micro instruction format and write a microprogram for the instruction 10  
 $ADD R_1, R_2$
- Q4 A Consider a 4-way set associative mapped cache with block size 4 KB. The size of the 10  
 main memory is 16 GB and there are 10 bits in the tag. Find-
1. Size of cache memory  
 2. Tag directory size
- B Explain Flynn's classification 10
- Q5 A Explain different types Distributed and Centralized bus arbitration methods 10
- B Describe the detailed Von-Neumann Model with a neat block diagram 05
- C Describe the characteristics of Memory. 05
- Q6 Write Short notes on 20
- a) Grey code, BCD, Excess-3 Code with example  
 b) Encoder and Decoder  
 c) Cache coherence  
 d) Instruction Pipelining

(3 Hours)

Total Marks: 80

**N.B: (1) Question No. 1 is compulsory.**

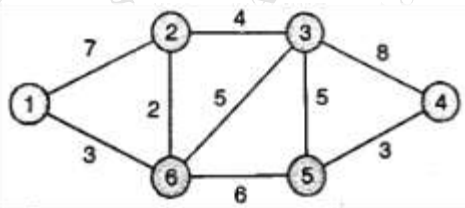
**(2) Attempt any three from the remaining questions.**

**(3) Figures to the right indicate full marks.**

1. Attempt any four
  - (a) Explain Best Case, Average Case and Worst Case. (05)
  - (b) Explain Multistage graphs. (05)
  - (c) Explain Binary search algorithm. (05)
  - (d) Define NP Class, NP hard, NP complete. (05)
  - (e) What is greedy algorithm? (05)
  
2.
  - (a) Write and explain sum of subset algorithm for  $n=5$ ,  $W = \{2,7,8,9,15\}$ ,  $M=17$ . (10)
  - (b) Obtain the solution to the following knapsack problem using Greedy method:  $n=7$ ,  $m=15$   
 $(p_1, p_2, \dots, p_7) = (10, 5, 15, 7, 6, 18, 3)$ ,  $(w_1, w_2, \dots, w_7) = (2, 3, 5, 7, 1, 4, 1)$ . (10)
  
3.
  - (a) What is the Longest Common Subsequence problem? Find the LCS for following strings (10)  
 String 1- ACBAED  
 String 2- ABCABE
  - (b) Explain quick sort with algorithm and example. (10)
  
4.
  - (a) What is Knuth Morris Pratt Method of Pattern Matching? Give Examples. (10)
  - (b) Solve the following Recurrence using Substitution Method. (10)

$$T(n) = \begin{cases} 1, & \text{if } n=1 \\ 2T(n/2) + Cn, & \text{if } n>1 \end{cases}$$

5.
  - (a) Find the Dijkstra's shortest path from vertex 1 to vertex 4 for the following graph. (10)



- (b) Apply Merge sort algorithm to sort the following numbers. Show each step clearly. 10, 5, 7, 6, 1, 4, 8, 3, 2, 9. (10)

6. Write notes on **(any two)**: (20)
  - (a) Find Minimum and Maximum elements of an array  $X[0 : 9] = (45, 83, 75, 17, 43, 37, 80, 53, 61, 22)$  using divide and conquer strategy.
  - (b) Naïve string matching algorithm with example.
  - (c) N-queen problem algorithm with example.

(3 Hours)

[Total Marks : 80]

N.B.

- 1) **Q.1 is compulsory.**
- 2) Solve any 3 questions out of remaining 5 questions.
- 3) Assumptions made should be clearly stated.
- 4) Draw the figures wherever required.

**Q.1 Solve any four of the following questions.**

- a) Prove using Mathematical Induction that  $n^3+2n$  is divisible by 3 for all  $n \geq 1$  5
- b) Explain the following terms with suitable example: 5
  - i) Partition set
  - ii) Power set.
- c) State the Pigeonhole principle and show that if any five numbers from 1 to 8 are chosen, then two of them will add to 9. 5
- d) Consider the function  $f(x) = 2x-3$ . Find a formula for the composition functions 5
  - i)  $f^2 = f \circ f$
  - ii)  $f^3 = f \circ f \circ f$
- e) Explain the bipartite graph with suitable example. 5

**Q.2**

- a) What is a transitive closure? Find the transitive closure of R using Warshall's algorithm where  $A = \{1, 2, 3, 4, 5\}$  &  $R = \{(x,y) \mid x-y = \pm 1\}$  10
- b) What is a ring? Let  $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ . Determine whether a set A with addition modulo 8 & multiplication modulo 8 is a commutative ring? Justify your answer. 10

**Q.3**

- a) A survey in 1986 asked households whether they had a VCR, a CD player or cable TV. 40 had a VCR. 60 had a CD player; and 50 had cable TV. 25 owned VCR and CD player. 30 owned a CD player and had cable TV. 35 owned a VCR and had cable TV. 10 households had all three. How many households had at least one of the three? How many of them had only CD player? 8
- b) Find the complete solution of a recurrence relation 6

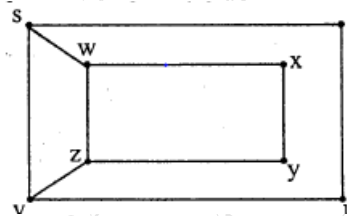
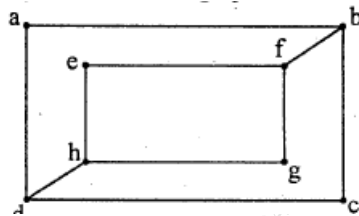
$$a_n + 2a_{n-1} = n + 3 \text{ for } n \geq 1 \text{ and with } a_0 = 3$$
- c) Obtain CNF & DNF for the following expression: 6

$$p \leftrightarrow (\sim p \vee \sim q)$$



**Q.4**

- a) What is a group? Let  $A = \{3, 6, 9, 12\}$  **10**  
 i) Prepare the composition table w.r.t. the operation of multiplication modulo 15.  
 ii) Whether it is an abelian group? Justify your answer.  
 iii) Find the inverses of all the elements.  
 iv) Whether it is a cyclic group?
- b) What are the isomorphic graphs? Determine whether following graphs are isomorphic. **10**



**Q.5**

- a) Let  $X = \{1, 2, 3, 6, 24, 36\}$  &  $R = \{(x,y) \in X \mid x \text{ divides } y\}$  **10**  
 i) Write the pairs in a relation set R.  
 ii) Construct the Hasse diagram.  
 iii) What are the Maximal and Minimal elements?  
 iv) Mention Chains and Ant chains from above set.  
 v) Is this poset a lattice?

- b) Define the term bijective function. **5**

Let  $f : \mathbb{R} \rightarrow \mathbb{R} - \left\{ \frac{2}{5} \right\} \rightarrow \mathbb{R} - \left\{ \frac{2}{5} \right\}$  be defined by  $f(x) = \frac{2x - 3}{5x - 7}$ .

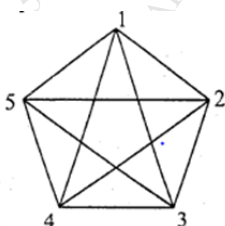
Whether a function is bijective? Justify your answer.

- c) Define minimum hamming distance. Consider  $e : B^3 \rightarrow B^6$ . Find the code words generated by the parity check matrix H given below. **5**

$$H = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

**Q.6**

- a) Define with example Euler path, Euler circuit, Hamiltonian path, and Hamiltonian circuit. Determine if the following diagram has Euler circuit and Hamiltonian circuit. Mention the path/circuit. **6**





- b) Let  $p$  denote the statement 'The food is good',  
 $q$  denote the statement 'The service is good' &  
 $r$  denote the statement 'The rating is 3 star.'

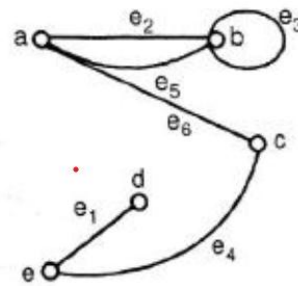
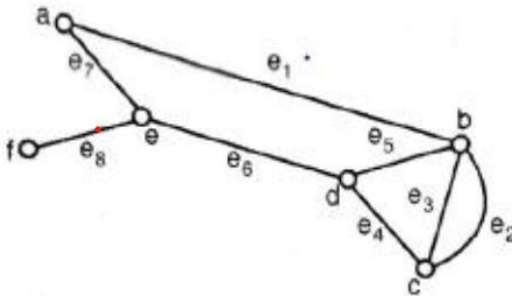
Write the following statements in a symbolic form-

- i) Either food is good or service is good or both.
- ii) The food is good but service is not good.
- iii) If both food & service are good then the rating is 3 star.
- iv) It is not true that a 3 star rating always means good food & good service.

8

- c) Find out the incidence matrix of following graphs.

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