

# University of Mumbai

Program: **Artificial Intelligence & Data Science**

Curriculum Scheme: Rev2019

Examination: SE Semester :III

Course Code: CSC301

Course Name: Engineering Mathematics - III

Time: 2 hour 30 minutes

Max. Marks: 80

<b>Q1.</b>	<b>Choose the correct option for following questions. All the Questions are compulsory and carry equal marks</b>
1.	Find $L[\sin^2 t]$
Option A:	$\frac{1}{2} \left[ \frac{1}{s} - \frac{s}{s^2 + 4} \right]$
Option B:	$\frac{s}{s^2 + 4}$
Option C:	$\frac{1}{2} \frac{s}{s^2 + 4}$
Option D:	$\frac{1}{s}$
2.	Find $L[e^t \cos t]$
Option A:	$\frac{1}{(s + 2)^2 + 5}$
Option B:	$\frac{2}{(s + 1)^2}$
Option C:	$\frac{s}{(s - 1)^2 + 1}$
Option D:	$\frac{2}{s}$

3.	Find $L^{-1}\left[\frac{s}{s^2 + 4}\right]$
Option A:	$e^{-t} \sin 2t$
Option B:	$\cos 2t$
Option C:	$\sin 2t$
Option D:	$e^{-t}$
4.	Find $L^{-1}\left[\frac{1}{s + 5}\right]$
Option A:	$(1 - e^{25t})$
Option B:	$e^{-5t}$
Option C:	$1 - e^{-5t}$
Option D:	$e^{-5t} / 5$
5.	Find half range sine series for $f(x) = x$ in $(0, \pi)$
Option A:	$-\sum_{n=1}^{\infty} \frac{2(-1)^n}{n} \sin nx$
Option B:	$\sum_{n=1}^{\infty} \frac{1 - (-1)^n}{2n} \cos nx$
Option C:	$\sum_{n=1}^{\infty} \frac{1 - (-1)^n}{2n}$
Option D:	$\sum_{n=1}^{\infty} \cos nx$
6.	Which of the following function is odd?
Option A:	$f(x) = x^2$
Option B:	$f(x) = x^2 - x$
Option C:	$f(x) = x$
Option D:	$f(x) = x^3 + x$
7.	Find the analytic function whose real part is $x^3 - 3xy^2$
Option A:	$z^3 + c$

Option B:	$z + c$								
Option C:	$z - c$								
Option D:	$3z + c$								
8.	Is $\phi(x) = x^2 - y^2 - 5x + y + 2$ Harmonic function								
Option A:	Yes								
Option B:	No								
Option C:	Sometimes Yes								
Option D:	Sometimes No								
9.	The Spearman's Rank Correlation Coefficient is lies in between								
Option A:	2 to -2								
Option B:	1 to -1								
Option C:	0 to 1								
Option D:	4 to 0								
10.	Find $P(X < 2)$ if proability distribution of X is								
	<table border="1"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>P(X)</td> <td>1</td> <td>3</td> <td>5</td> </tr> </table>	X	0	1	2	P(X)	1	3	5
X	0	1	2						
P(X)	1	3	5						
Option A:	3								
Option B:	4								
Option C:	2								
Option D:	1								

<b>Q2</b>	<b>Solve any Four Questions out of Six</b> <b>5 marks each</b>												
A	Find $L^{-1} \left[ \frac{s^2}{(s+a)^2} \right]$ using convolution theorem												
B	Find Fourier series for $f(x) = x^2$ in $(0, 2\pi)$												
C	Show that $u = x^3y - xy^3$ is a harmonic function. Find its harmonic conjugate and analytic function												
D	A continuous random variable $X$ has the probability density function $f(x) = kx^2, 0 \leq x \leq 2$ . Determine $k$												
E	Compute Karl Pearson's coefficient of correlation between X & Y												
	<table border="1"> <tr> <td>X</td> <td>3</td> <td>6</td> <td>4</td> <td>5</td> <td>7</td> </tr> <tr> <td>Y</td> <td>2</td> <td>4</td> <td>5</td> <td>3</td> <td>6</td> </tr> </table>	X	3	6	4	5	7	Y	2	4	5	3	6
X	3	6	4	5	7								
Y	2	4	5	3	6								

F	Find $L\left[\int_0^t \sin u \, du\right]$
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Q3.	Solve any Four Questions out of Six	5 marks each
A	Find the Laplace transform of $\cos t \cdot \cos 2t \cdot \cos 3t$	
B	Using convolution theorem find the Inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)^2}$	
C	Find the Fourier expansion of $f(x) = x + x^2$ ; $-\pi \leq x \leq \pi$ and $f(x + 2\pi) = f(x)$	
D	Find $k$ & then $E(X)$ , if $X$ has the probability density function $f(x) = \begin{cases} kx(2-x), & 0 \leq x \leq 2, k > 0 \\ 0, & \text{otherwise} \end{cases}$	
E	Find an analytic function $f(z)$ whose imaginary part is $e^{-x}(y \sin y + x \cos y)$	
F	Obtain the rank correlation coefficient from the following data $X : 10, 12, 18, 18, 15, 40$ $Y : 12, 18, 25, 25, 50, 25$	

Q4.	Solve any Four Questions out of Six	5 marks each
A	By using Laplace transform, evaluate $\int_0^{\infty} e^{-t} \left( \frac{\cos 3t - \cos 2t}{t} \right) dt$	
B	Find the inverse Laplace transform of $\tan^{-1} \left( \frac{2}{s^2} \right)$	
C	Find the orthogonal trajectory of the family of curves $x^3 y - xy^3 = c$	

D	<p>A random variable <math>X</math> has the following probability function</p> $  \begin{array}{ccccccc}  X & : & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\  P(X = x) & : & k & 2k & 3k & k^2 & k^2 + k & 2k^2 & 4k^2  \end{array}  $ <p>Find i) <math>k</math> and ii) <math>P(X &lt; 5)</math></p>
E	<p>Obtain the expansion of <math>f(x) = x(\pi - x)</math>; <math>0 &lt; x &lt; \pi</math> as a half-range cosine series.</p>
F	<p>Fit a straight line of the form <math>y = a + bx</math> to the following data &amp; estimate the value of <math>y</math> for <math>x = 3.5</math></p> $  \begin{array}{cccccc}  x & : & 0 & 1 & 2 & 3 & 4 \\  y & : & 1 & 1.8 & 3.3 & 4.5 & 6.3  \end{array}  $